Data-Driven Model Order Reduction for Magnetostatic Problem Coupled with Circuit Equations

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Among the model order reduction techniques, the Proper Orthogonal Decomposition (POD) has shown its efficiency to solve magnetostatic and magneto-quasistatic problems in the time domain. However, the POD is intrusive in the sense that it requires the extraction of the matrix system of the full model to build the reduced model. Then, nonintrusive approaches like the Data Driven (DD) method enables to approximate the reduced model without the access of the full matrix system. In this communication, the DD method is applied to solve a magnetostatic problem coupled with electric circuit equations.

***Index Terms*—Data driven, finite element model, model order reduction.**

I. Introduction

To reduce the computation time of large-scale dynamical Finite Element (FE) models, Model Order Reduction (MOR) methods have been developed and presented in the literature. These methods consist in searching a solution into an approximation subspace of the full numerical model. Then, the size of the equation system to solve can be highly reduced. The most popular MOR technique is the Proper Orthogonal Decomposition (POD) [1]. This approach requires to solve the full model for different time steps (called snapshots) to determine a reduced basis. Then, from the matrix system of the full model, the reduced model can be constructed and solved for all other time steps. This approach is well adapted if the matrix system can be extracted from the FE software. With commercial FE softwares, the matrix system is not necessarily accessible. Nevertheless, nonintrusive approaches of MOR, like Data Driven (DD) methods, have been developed in the literature [2]. One of these approaches [3], based on the snapshots, builds an approximation of the reduced model from the known inputs and outputs of the full model. In low frequency, a significant number of nonlinear magnetostatic or magnetodynamic problems have been studied with POD but not with nonintrusive approaches [4], [5], [6], [7].

In this communication, the DD approach is applied to build a reduced model in order to solve a magnetostatic problem coupled with electric circuit equations using the vector potential formulation. The DD reduced model is based on the known inputs (voltage, current, resistances, ...) and the outputs (linkage flux, solution vector, ...) from the snapshots. A three phase transformer is studied with the classical POD method and the DD approach. The results obtained with both reduced models are compared in terms of accuracy and computation time with the full model.

II. Magnetostatic problem with electric equations

To solve a magnetostatic problem coupled with N_{ind} electric circuit equations, the vector potential formulation can be used. After discretisation by the FE method, the matrix system to solve is

$$\begin{bmatrix} 0 & 0 \\ F^t & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{i}} \end{bmatrix} + \begin{bmatrix} M & F \\ 0 & R \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{i} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{v} \end{bmatrix}, \qquad (1)$$

with M the stiffness matrix depending on the magnetic reluctivity, F a matrix depending on the geometries of the stranded inductors, $\mathbf{x}$ the vector solution of size N_x composed of the circulations of the vector potential A along the edges of the mesh, $\mathbf{i}$ the currents flowing the stranded inductors, $\mathbf{v}$ the voltages applied to the stranded inductor terminals and R the diagonal matrix of the resistances. The linkage flux ϕ_k associated with the k-th inductor is expressed by $\phi_k = F_k^t \mathbf{x}$.

III. Model order reduction

A. Proper Orthogonal Decomposition

To conserve the structure of the matrix system during the reduction and insure stability, a structure preserving MOR [6] is performed. The full model (1) is solved for different time steps in order to obtain a set of N_s solutions and to create the snapshot matrix X_s such as $X_s = [\mathbf{x}(t_1), \mathbf{x}(t_2), \ldots, \mathbf{x}(t_{N_s})] \in \mathbf{R}^{N_x \times N_s}$. The snapshots can be determined during the first time steps or in a preprocessing step based on a greedy algorithm. The singular value decomposition applied to X_s, such as $X_s = U \Sigma W^t$, allows to obtain the reduced basis $\Psi \in \mathbf{R}^{N_x \times N_r}$ formed by the N_r ($N_r \leq N_s$) first columns of U. N_r can be determined by a truncation strategy. The solution vector $\mathbf{x}$ is approximated with the basis Ψ such that $\mathbf{x} \simeq \Psi \mathbf{x}_r$, and then the equation associated with the magnetic part in (1) is multiplied by Ψ^t to obtain the reduced system,

$$\begin{bmatrix} 0 & 0 \\ F_r^t & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}}_r \\ \dot{\mathbf{i}} \end{bmatrix} + \begin{bmatrix} M_r & F_r \\ 0 & R \end{bmatrix} \begin{bmatrix} \mathbf{x}_r \\ \mathbf{i} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{v} \end{bmatrix} \qquad (2)$$

with $F_r = \Psi^t F$ and $M_r = \Psi^t M \Psi$.

B. Data-Driven POD

The principle of the DD combined with the POD is to create an approximation of the reduced system (2) from the snapshots of the full model, without knowing the full matrix system.

Snapshot matrices associated with each output are defined such as $X_s = [\mathbf{x}(t_1), \mathbf{x}(t_2), \ldots, \mathbf{x}(t_{N_s})] \in \mathbf{R}^{N_x \times N_s}$, $I_s = [\mathbf{i}(t_1), \mathbf{i}(t_2), \ldots, \mathbf{i}(t_{N_s})]$ and $\Phi_s = [\phi(t_1), \phi(t_2), \ldots, \phi(t_{N_s})] \in \mathbf{R}^{N_{\text{ind}} \times N_s}$. From X_s, a reduced basis Ψ is determined in the same way as the one used for the POD (see III-A), and the X_s is projected onto the reduced space by $X_r = \Psi^t X_s$. The idea of the DD-POD is to find the reduced operators (F_r and M_r) of the system (2) from I_s, Φ_s and X_r. Firstly, the operator F_r is deduced by using the relation between the solutions and the fluxes that must be verified for each snapshot: then we have $X_r^t F_r = \Phi_s^t$. Each column of F_r can be identified by solving the minimisation problem

$$F_{r,k} = \arg\min_y \|X_r^t y - \Phi_{s,k}^t\|, \ k = 1, \ldots, N_{\text{ind}}. \quad (3)$$

Secondly, the matrix M_r is deduced from the first equation of (2). This relation must be verified for each snapshot such that $X_r^t M_r^t + I_s^t F_r^t = 0$. Then, the previous system is overdetermined. The k-th row of M_r can be determined by

$$M_{r,k}^t = \arg\min_y \|X_r^t y - I_{s,k}^t F_{r,k}^t\|, \ k = 1, \ldots, N_r. \quad (4)$$

IV. Application

In term of application, a linear 2D three-phase EI transformer is considered. Figure 1 presents the mesh. To built the reduced models from the POD and DD-POD approaches, the Offline/Online method is used. During the Offline step, the snapshots are computed for the typical tests at no load and in short circuit [7] on the first period, with 30 time-steps. Then, the POD and DD-POD models are defined and used to study another operating point. The full model has 4265 spatial unknowns, when both reduced models are reduced to 5 unknowns. During the Online step, a resistive load is connected with the secondary windings. Figure 2 presents the evolutions of the primary currents at the beginning of the simulation obtained from the full, POD and DD-POD models. The waveform of the currents from both reduced models are close to the references. The mean error on the current is 0.4013% with the DD-POD model and 0.0001% with the POD model. For both models, the speed-up is close to 250. Figure 3 gives the distribution of the magnetic flux density from the full model at a given time step and the errors between the full model and the reduced models. The errors are smaller with the POD model than the DD-POD model, nevertheless the magnitudes of the error from the DD-POD are small compared with the magnetic flux density.

The efficiency of the DD-POD approach with a nonlinear behavior of the magnetic core will be investigated.

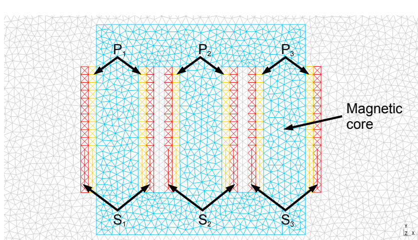

Fig. 1. Mesh of the 2D three-phase transformer.

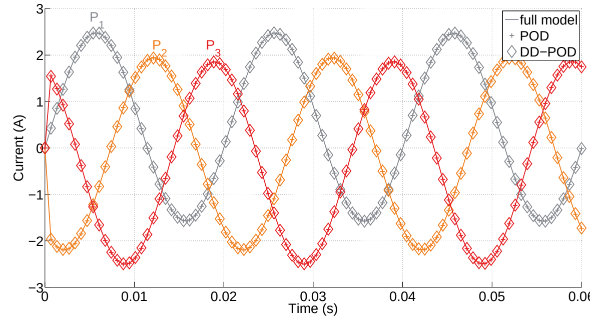

Fig. 2. Primary currents for the reference, POD and DD-POD models.

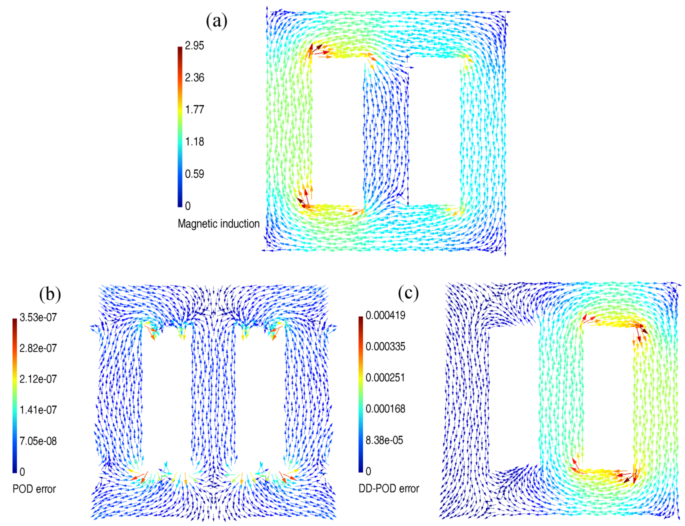

Fig. 3. Distribution of the magnetic flux density (T) from the full model (a) and of the error for the POD (b) and DD-POD (c) models.

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